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THESIS

PERFORMANCE OF COHERENT MULTILEVEL DIGITAL COMMUNICATIONS RECEIVERS IN THE PRESENCE OF NOISE AND JAMMING

by

Armando J. Mazzotti Vila

March 1985

Thesis Advisor: Daniel Bukofzer

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Performance of Coherent Multilevel Digital Communications Receivers in the Presence of Noise and Jamming

by

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requirements for the degree of

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March 1985

ABSTRACT

The effect of jamming waveforms on optimum multilevel digital coherent communications receivers designed to operate in a Gaussian noise only environment is analyzed and evaluated in terms of receiver performance. Near optimum jamming waveforms (such as a tone jammer and a weighted sum of signals jammer) are postulated in order to determine their effect on the performance of an M-ary Phase Shift Keying coherent receiver. Additionally, the optimum power constrained jamming waveform is derived and analyzed for an M-ary Amplitude Shift Keying coherent receiver. Graphical results of numerical analyses resulting from the evaluation of receiver performance are presented and interpreted in order to quantify the effectiveness of the jammers. Receiver performance is measured in terms of word error probability as a function of signal-to-noise ratio.

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I. INTRODUCTION

The objective of this thesis is to determine the effect of deterministic jamming waveforms on the performance in terms of word error probability (P_e) of multilevel digital coherent communications receivers which are designed to operate in an additive white Gaussian noise (AWGN) environment.

Specific mathematical models of signals are utilized and jamming waveforms are postulated in order to determine receiver performance.

The analysis and results are presented in three sections. In the first section, optimum M-ary receiver structures are presented. These are well-known structures that can be derived using decision theory hypothesis testing concepts. Two specific cases of signaling or modulation techniques are analyzed, namely M-ary Phase Shift Keying (M-PSK) and M-ary Amplitude Shift Keying (M-ASK) in AWGN only interference. Results on word error probability for the M-PSK and M-ASK receivers are presented in mathematical form. In the second section, performance of coherent M-PSK and M-ASK receivers presented in the first section are analyzed in the presence of near optimum and optimum jamming waveforms respectively. The jamming waveforms postulated and analyzed are a weighted sum of signals jammer and a pure tone jammer with set phase. These are evaluated in terms of their effect on P_e for the M-PSK receiver. An optimum jamming waveform to be used against an M-ASK receiver, and based on jammer power constraints is derived and analyzed in this section also. The problem of jamming multilevel Frequency Shift Keying (FSK) receivers has been analyzed in [Ref. 1]. The determination of the effect of the jammer on the receiver is made simpler by the fact that generally, the M-FSK signals form an orthogonal set. Finally, the third section presents graphical results obtained from the evaluation of performance expressions derived in Section 2. The presented plots of P_e versus signal to noise ratio (SNR) are used in order to quantify the effectiveness of the jammers.

II. M-ARY COHERENT RECEIVER ANALYSIS

A. RECEIVER STRUCTURE

For the implementation of a multilevel digital communications receiver, the derivation of the system for the recovery of the transmitted signal starts from the assumption that r(t), the signal appearing at the front end of the receiver can be mathematically modeled by

$$r(t) = s_i(t) + n(t) \quad t_o \leq t \leq t_f \quad i=0,1,\ldots,M-1 \quad (2.1)$$

where $s_i(t)$ is one of the M possible signals used to transmit the information and n(t) is a sample function of a white Gaussian noise (WGN) process having a two sided power spectral density (PSD) level $N_o/2$ watts/Hz.

The optimum receiver for deciding which of the M signals was transmitted in the interval $[t_o, t_f]$ (with minimum probability of error) is well-known and its structure is detailed in [Ref. 2]. An alternative receiver implementation [Ref. 3] shown in Figure 1, is derived using decision theory hypothesis testing concepts. That is, the observations are set up as an M-ary hypotheses testing problem [Ref. 4].

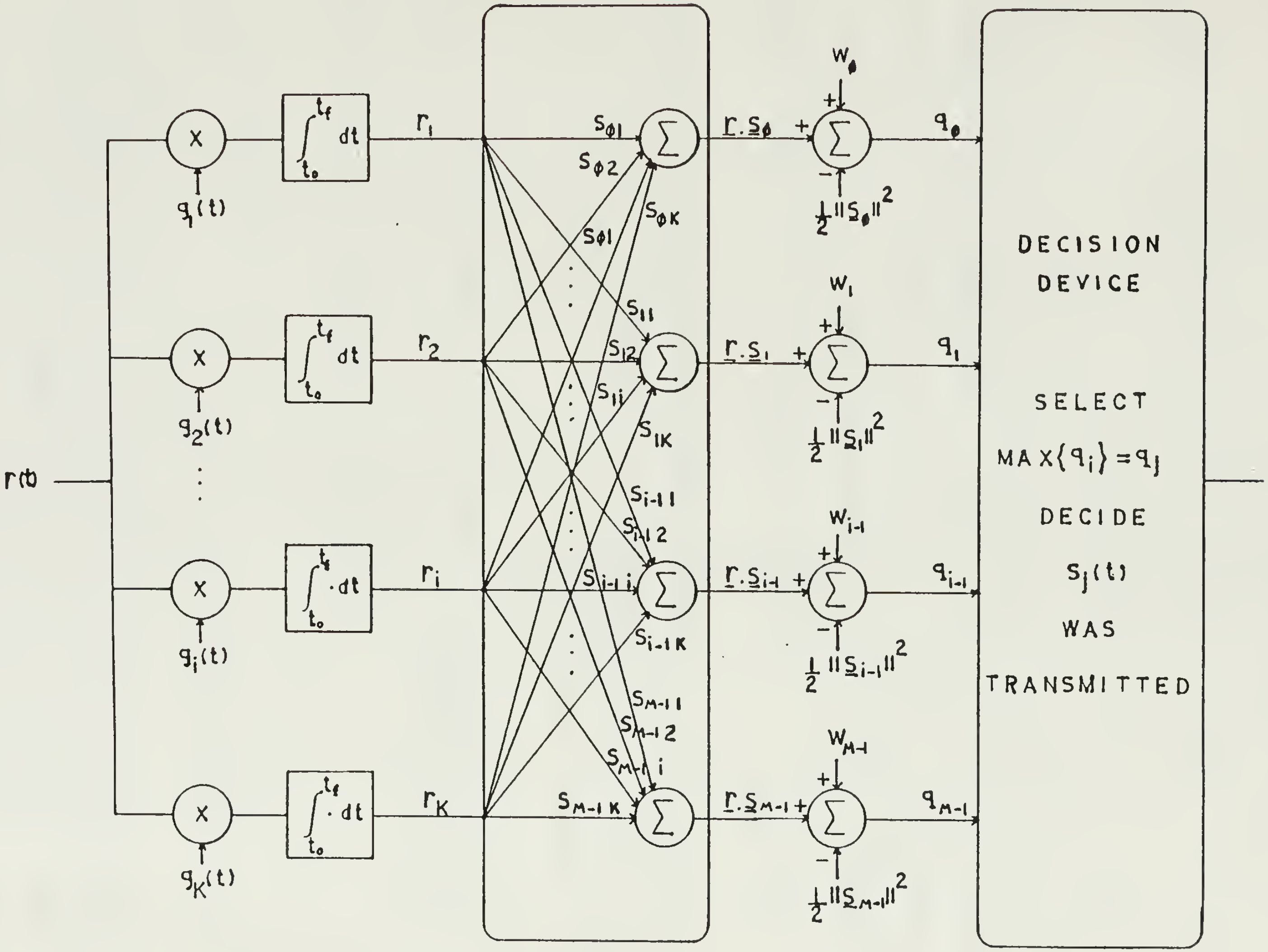

Figure 1 M-ary Coherent Receiver Structure

The M hypotheses are expressed as :

$$\begin{array}{lll} H_0 & : & r(t) = s_0(t) + n(t) \\ H_1 & : & r(t) = s_1(t) + n(t) \qquad t_o \leq t \leq t_f \\ \vdots & & \vdots \\ H_{M-1} & : & r(t) = s_{M-1}(t) + n(t) \end{array} \tag{2.2}$$

The detection algorithm or equivalently, the receiver structures shown are the result of the following procedure. The observed waveform is expanded in terms of a complete orthonormal (CON) set of basis functions

$$\{g_k(t)\}_{k=1}^{\infty} \qquad t_o \leq t \leq t_f \tag{2.3}$$

where the first $K \leq M$ basis functions are derived from the signals $s_i(t)$ via (perhaps) a Gram-Schmidt orthonormalization procedure, so that

$$r(t) = \sum_{k=1}^{K} r_k g_k(t) \tag{2.4}$$

where

$$r_k = \int_{t_o}^{t_f} r(t)\, g_k(t)\, dt = s_{ik} + n_k \tag{2.5}$$

The second equality has been obtained on the assumption that the signal $s_i(t)$ was transmitted. Observe that

$$s_i(t) = \sum_{k=1}^{K} s_{ik}\, g_k(t) \tag{2.6}$$

where

$$s_{ik} = \int_{t_0}^{t_f} s_i(t) g_k(t) dt \qquad \begin{matrix} i=0,1,\ldots,M-1 \\ k=1,2,\ldots,K \end{matrix} \tag{2.7}$$

also

$$n_k = \int_{t_0}^{t_f} n(t) g_k(t) dt \tag{2.8}$$

The decision rule implemented is to choose H_i as the true hypothesis if the quantity q_i, where

$$\begin{aligned} q_i &= \underline{r}\cdot\underline{s}_i + w_i - \frac{1}{2} ||\underline{s}_i||^2 \\ &= \sum_{k=1}^{K} r_k s_{ik} + w_i - \frac{1}{2} \sum_{k=1}^{K} s_{ik}^2 \end{aligned} \tag{2.9}$$

is the largest. The multiplication of two vectors is to be interpreted as a dot or scalar product, $||.||$ stands for norm or vector length, w_i is defined as

$$w_i = \frac{N_o}{2} \ln P\{H_i\} \tag{2.10}$$

and $\underline{r}$ and $\underline{s}_i$ are the (column) vectors

$$\underline{r} = [r_1 \; r_2 \; \ldots \; r_K]^T \tag{2.11}$$

$$\underline{s}_i = [s_{i1} \; s_{i2} \; \ldots \; s_{iK}]^T \tag{2.12}$$

The prior probabilities of occurence of the ith hypothesis is denoted $P\{H_i\}$ for $i=0,1,\ldots,M-1$. The receiver structure implementing this decision rule is diagramed in Figure 1.

Further simplifications to the receiver structure can be obtained if additional assumptions are made.

Two specific cases of signaling (or modulation) techniques will be analyzed. These are M-ary Phase Shift Keying (M-PSK) and M-ary Amplitude Shift Keying (M-ASK). As will be demonstrated, receiver simplifications result in such a way that for M-PSK only two correlators are needed and for M-ASK just one correlator is required in the receiver structure.

1. M-PSK Coherent Receiver Structure

For this type of signaling (or modulation) each signal has a different phase and can be written as

$$s_i(t)=A\cos(w_c t + 2\pi i/M) \quad t_o \leq t \leq t_f,\ i=0,1,\ldots M-1 \tag{2.13}$$

We will assume for mathematical simplicity that

$$w_c\,(t_f - t_o) = n\pi \tag{2.14}$$

where n is some integer.

Determination of the basis functions $g_k(t)$ previously described can be accomplished using the Gram-Schmidt procedure [Ref. 5], or more simply by observing that

$$s_i(t) = A \cos w_c t \cos \theta_i - A \sin w_c t \sin \theta_i \qquad (2.15)$$

where

$$\theta_i = \frac{2\pi i}{M} \qquad i=0,1,\ldots,M-1 \qquad (2.16)$$

Due to the assumption of Equation 2.14, cos $w_c t$ and sin $w_c t$ are orthogonal functions over the interval $[t_o , t_f]$ so that a simple normalization is required to arrive at a general expression for the signals $s_i(t)$ written in terms of the basis functions. Thus,

$$g_1(t) = \frac{\cos w_c t}{\sqrt{(t_f-t_o)/2}} \qquad (2.17)$$

and

$$g_2(t) = \frac{\sin w_c t}{\sqrt{(t_f-t_o)/2}} \qquad (2.18)$$

so that Equation 2.15 becomes

$$s_i(t)=A\sqrt{(t_f-t_o)/2}\ \cos\theta_i\ g_1(t) + \left(-A\sqrt{(t_f-t_o)/2}\right)\sin\theta_i\ g_2(t) \qquad (2.19)$$

By inspection, the components of $s_i(t)$ in the direction of $g_1(t)$ and $g_2(t)$ are respectively

$$s_{i1} = A\sqrt{(t_f-t_o)/2}\ \cos\theta_i \qquad (2.20)$$

and

$$s_{i2} = -A\sqrt{(t_f-t_o)/2}\,\sin\theta_i \tag{2.21}$$

The cross correlation of any pair of these signals is

$$\rho_{ij} = \int_{t_o}^{t_f} s_i(t)\, s_j(t)\, dt = \frac{A^2}{2}(t_f - t_o)\cos 2\pi(i-j)/M \tag{2.22}$$

Observe that the energy of each signal is

$$E_i = \int_{t_o}^{t_f} s_i^2(t)\, dt \doteq ||\underline{s}_i||^2 = A^2(t_f - t_o)/2 \doteq E \qquad i=0,1,\ldots,M-1 \tag{2.23}$$

where $\underline{s}_i$ is the vector of components s_{i1} and s_{i2}. Thus Equation 2.23 demonstrates that all signals have equal energy and therefore a normalized cross correlation can be defined as

$$\overline{\rho}_{ij} = \rho_{ij} / \sqrt{E_i}\sqrt{E_j} = \cos\ (2\pi(i-j)/M) \tag{2.24}$$

Assuming that all signals have the same probability of occurence, that is

$$P\{H_i\} = 1/M \qquad i=0,1,\ldots,M-1 \tag{2.25}$$

then from Equation 2.23 it follows that, the terms w_i and $\frac{1}{2}||\underline{s}_i||^2$ in the receiver of Figure 1 become independent of the index i. Thus, equal weighting is being used in each channel of the receiver of Figure 1. Such weighting is not necessary and can be eliminated. From Equation 2.9 and the above, it can be seen that the receiver need only obtain $\underline{r} \cdot \underline{s}_i$. Thus, the receiver actually computes q_i, where now

$$q_i = \underline{r} \cdot \underline{s}_i = \int_{t_0}^{t_f} r(t)\, s_i(t)dt \qquad i=0,1,\ldots,M-1 \quad (2.26)$$

and bases its decision on which q_i is largest.

Determination of performance of the receiver in terms of probability of error P_e will require either some modifications to the receiver structure, or a reinterpretation of the problem in such a way that polar coordinates can be used to represent the ith channel output q_i. That is, q_i is expressed in terms of its amplitude V and phase α [Ref. 6]. Using Equation 2.15 in Equation 2.26, q_i becomes

$$q_i = \cos\theta_i \int_{t_0}^{t_f} Ar(t)\cos w_c t\, dt - \sin\theta_i \int_{t_0}^{t_f} Ar(t)\sin w_c t\, dt$$

$$= V\cos(\theta_i + \alpha) \qquad (2.27)$$

where

$$V = \sqrt{V_c^2 + V_s^2} \qquad 0 \leq V < \infty \qquad (2.28)$$

$$V_c = V\cos\alpha = \int_{t_0}^{t_f} Ar(t)\cos w_c t\, dt \qquad (2.29)$$

$$V_s = V\sin\alpha = \int_{t_0}^{t_f} Ar(t)\sin w_c t\, dt \qquad (2.30)$$

and

$$\alpha = \tan^{-1} (V_s/V_c) \qquad 0 \leq \alpha \leq 2\pi \qquad (2.31)$$

Recall that the receiver decides which hypothesis is true based on which q_i is largest. If $s_m(t)$ is transmitted, a correct decision is made if $q_m > q_i$ for $i=0,1,\ldots,M-1$, $i \neq m$, or equivalently using Equation 2.27, if

$$V \cos (\theta_m + \alpha) > V \cos (\theta_i + \alpha) \qquad \text{for all } i \neq m \qquad (2.32)$$

This condition is satisfied if

$$| \theta_m + \alpha | < | \theta_i + \alpha | \qquad \text{for all } i \neq m \qquad (2.33)$$

Thus, this equivalent test or decision rule can be translated into the alternative receiver structure for M-PSK as shown in Figure 2. Observe that only two correlators are required for the computation of V_c and V_s from which the receiver detects the phase of the input signal.

2. M-ASK Coherent Receiver Structure

In M-ary Amplitude Shift Keying, each signal has a different amplitude and can be mathematically modeled as

$$s_i(t) = A_i \, f(t) \qquad i=0,1,\ldots,M-1 \ , \ t_o \leq t \leq t_f \qquad (2.34)$$

Let

$$E_f = \int_{t_o}^{t_f} f^2(t)dt \qquad (2.35)$$

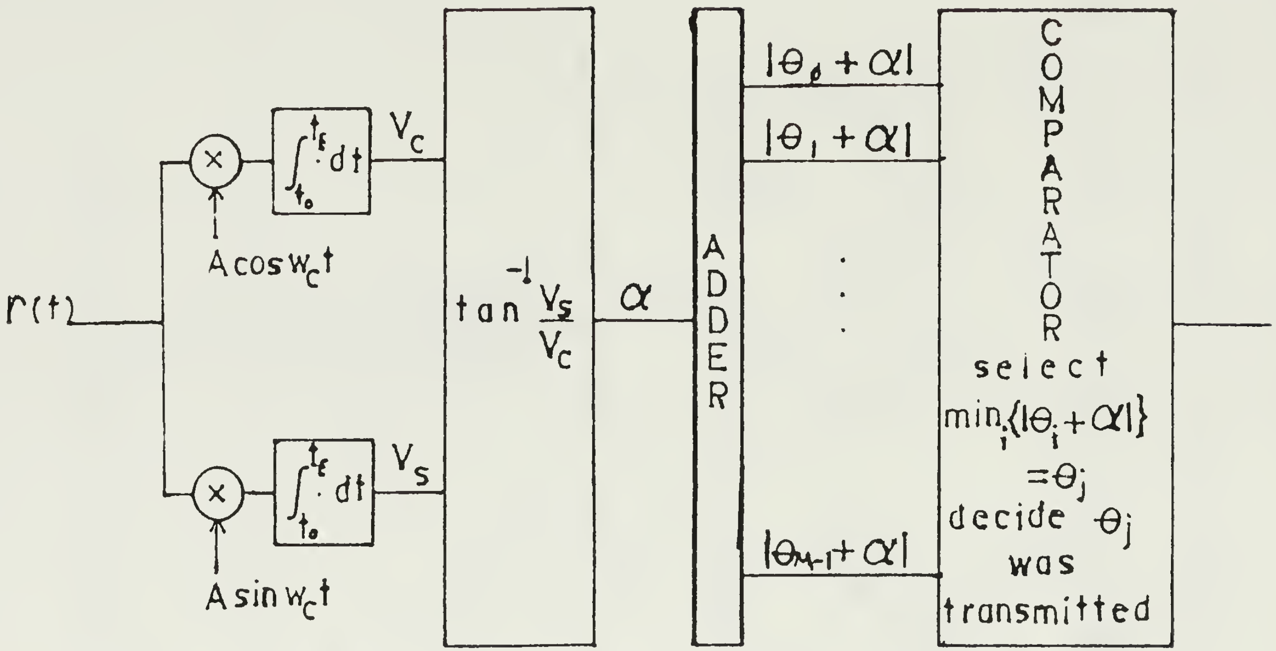

Figure 2 M-PSK Coherent Receiver Structure

so that only one basis function is needed to represent the signals $s_i(t)$, namely

$$g_1(t) = f(t)/\sqrt{E_f} \qquad t_o \leq t \leq t_f \tag{2.36}$$

and therefore

$$s_i(t) = A_i \sqrt{E_f}\, g_1(t) \qquad i=0,1,\ldots,M-1 \tag{2.37}$$

Thus, the component of the signal $s_i(t)$ in the direction of $g_1(t)$ is

$$s_{i1} = A_i \sqrt{E_f} \qquad i=0,1,\ldots,M-1 \tag{2.38}$$

and the energy of each signal is

$$||\underline{s}_i||^2 = E_i = \int_{t_o}^{t_f} s_i^2(t)\, dt = A_i^2 E_f \tag{2.39}$$

We will assume that all signals have the same probability of occurence. Furthermore, since only one basis function is needed to represent the signals $s_i(t)$, $i=0,1,\ldots,M-1$, the receiver requires only one correlator so that the receiver structure takes on the form shown in Figure 3. From the figure, it can be seen that the output of each channel is

$$q_i = \left[\int_{t_o}^{t_f} r(t) g_1(t) dt\right] A_i\sqrt{E_f} - \frac{1}{2} A_i^2 E_f \tag{2.40}$$

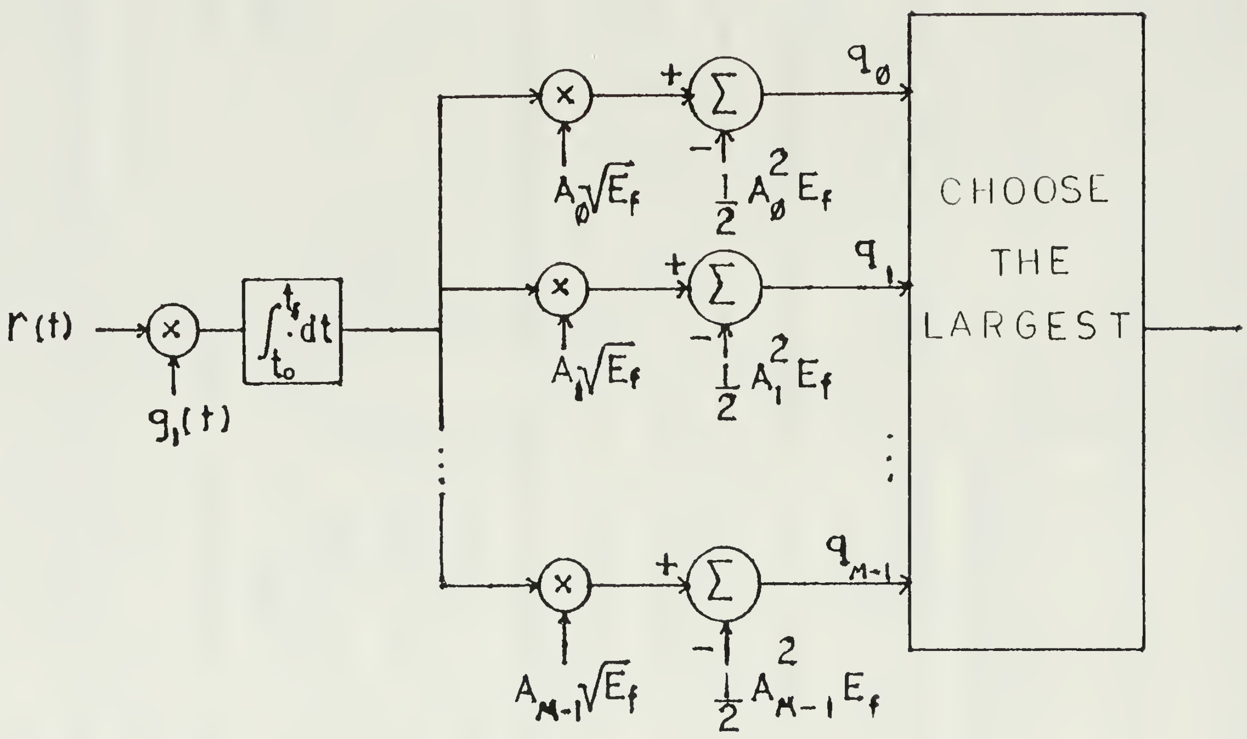

Figure 3 M-ASK Coherent Receiver Structure

and the receiver chooses H_j as the true hypothesis if q_j is the largest. That is, if

$$q_j > q_0,\ q_j > q_1,\ \ldots, q_j > q_{j-1},\ q_j > q_{j+1}, \ldots, q_j > q_{M-1} \qquad (2.41)$$

then a decision is made that $s_j(t)$ was the transmitted signal.

B. RECEIVER PERFORMANCE

Receiver performance in terms of word error probability P_e for M-ary coherent receivers operating in additive white Gaussian noise is well documented for different signal sets or modulation schemes [Ref. 2, 6]. The computation of P_e requires finding the probability that one Gaussian random variable q_m exceeds M-1 other q_i, $i \neq m$, Gaussian random variables which are jointly Gaussian but are not in general statistically independent. This tends to make the evaluation of P_e quite difficult in general. Many of the M-ary modulation schemes in practice lead to closed form mathematical expressions for P_e that are quite tractable. In the next subsection, results on P_e for M-PSK and M-ASK are presented. That is, word error probabilities for the receivers of Figure 2 and Figure 3 are presented in mathematical form. In the section following the present one, the effect on the performance of the receivers of Figure 2 and Figure 3 due to some deterministic jamming waveforms, that are unknown to the receiver itself, will be analyzed.

1. M-PSK Receiver Performance

The word error probability of the M-PSK coherent receiver, $(M \geq 2)$, operating in an additive white Gaussian noise environment [Ref. 3] is

$$P_e = 1 - \int_{-\pi/M}^{\pi/M} \int_0^{\infty} \exp[- \frac{1}{2}[r^2 - 2\, r \sqrt{SNR} \cos\beta + SNR]]\, dr d\beta \tag{2.42}$$

Introducing the following change of variables

$$u = r \cos\beta \tag{2.43}$$

and

$$v = r \sin\beta \tag{2.44}$$

then P_e can be re-expressed [Ref. 3] as

$$P_e = 1 - 2\int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(u - \sqrt{SNR})^2/2} \int_0^{\frac{u}{\sqrt{2}} \tan\frac{\pi}{M}} \frac{1}{\sqrt{2\pi}} e^{-v^2/2}\, dv du \tag{2.45}$$

where the signal to noise ratio is defined as

$$SNR = \frac{A^2\, (t_f - t_o)\ /2}{N_o/2} = \frac{E}{N_o/2} \tag{2.46}$$

and E is the symbol energy.

Note that if M=2, Equation 2.42 simplifies to yield the word error probability of the Binary Phase Shift Keying (BPSK) receiver which in this case is equivalent to the bit error probability given by [Ref. 3].

$$P_e = \mathrm{erfc}_*\left(\sqrt{SNR}\right) \qquad (2.47)$$

The complementary error function used throughout this thesis is defined by

$$\mathrm{erfc}_*(a) = \int_a^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx \qquad (2.48)$$

For M=4 (QPSK), the word error probability of the QPSK receiver [Ref. 2] simplifies to

$$P_e = 2\,\mathrm{erfc}_*\left(\sqrt{\frac{SNR}{2}}\right) - \mathrm{erfc}_*^2\left(\sqrt{\frac{SNR}{2}}\right) \qquad (2.49)$$

For M > 2 and large values of SNR that guarantee $P_e < 10^{-3}$ [Ref. 6] the word error probability can be approximated by

$$P_e \cong 2\,\mathrm{erfc}_*\left(\sqrt{SNR}\,\sin\frac{\pi}{M}\right) \qquad (2.50)$$

2. M-ASK Receiver Performance

The word error probability of the M-ASK coherent receiver assuming that the M signals are ordered in increasing amplitude and that the separation between each pair of consecutive signals is constant, labeled A, [Ref. 3] is given by

$$P_e = 2\ \mathrm{erfc}_*\left(\frac{1}{2}\sqrt{SNR}\right) \tag{2.51}$$

The signal to noise ratio is defined here as

$$SNR = A^2\ E_f\ /\ (N_o/2) \tag{2.52}$$

where E_f is defined in Equation 2.35.

III. M-ARY COHERENT RECEIVER PERFORMANCE ANALYSIS IN THE PRESENCE OF JAMMING

The analysis proceeds now under the assumption that a jamming waveform J(t) is present in the transmission channel, so that at the front end of the receiver the signal r(t) takes on the form

$$r(t) = s_i(t) + n(t) + J(t) \qquad t_o \leq t \leq t_f, \quad i=0,1,\dots,M-1 \tag{3.1}$$

The receiver will therefore have to test and decide amongst the following M hypotheses :

$$\begin{array}{lll} H_0 & : & r(t) = s_0(t) + n(t) + J(t) \\ H_1 & : & r(t) = s_1(t) + n(t) + J(t) \\ \vdots & & \vdots \\ H_{M-1} & : & r(t) = s_{M-1}(t) + n(t) + J(t) \end{array} \qquad t_o \leq t \leq t_f \tag{3.2}$$

using the decision rule implemented by the structure of Figure 1. The jammer J(t) is modeled as deterministic yet unknown to the receiver. The analysis is therefore designed to determine the effect of the jammer on the receiver performance.

A. PERFORMANCE OF THE M-PSK RECEIVER IN JAMMING

In the presence of a deterministic jamming waveform J(t), the output of the correlators of the receiver of Figure 2 become

$$V_c = A \int_{t_o}^{t_f} [\ s_i(t)+n(t)+J(t)\ ] \cos w_c t \, dt = V \cos \alpha \tag{3.3}$$

and

$$V_s = A \int_{t_o}^{t_f} [\ s_i(t)+n(t)+J(t)\ ] \sin w_c t \, dt = V \sin \alpha \tag{3.4}$$

Observe that conditioned on any hypothesis, V_c and V_s are Gaussian random variables with expected value m_c and m_s respectively, where

$$m_c = E\,[V_c/H_i] = A\sqrt{(t_f - t_o)/2}\quad [s_{i1} + J_1] \qquad i=0,1,\dots,M-1 \tag{3.5}$$

and

$$m_s = E\,[V_s/H_i] = A\sqrt{(t_f - t_o)/2}\quad [s_{i2} + J_2] \qquad i=0,1,\dots,M-1 \tag{3.6}$$

where s_{i1} and s_{i2} are defined in Equation 2.20 and Equation 2.21 respectively and J_1 and J_2 are the components of the jamming waveform $J(t)$ in the direction of $g_1(t)$ and $g_2(t)$ respectively and defined by the inner product

$$J_k = \int_{t_o}^{t_f} J(t)\ g_k(t)\, dt \qquad k = 1,2 \tag{3.7}$$

Furthermore, the conditional variance of V_c and V_s can be shown to be

$$\mathrm{Var}\,[V_c/H_i] = \mathrm{Var}\,[V_s/H_i] \doteq \sigma^2 = \frac{A^2}{2}\,(t_f - t_o)\,.\,N_o/2$$

$$i=0,1,\ldots,M-1 \qquad (3.8)$$

and their conditional covariance is

$$E\,\{\,[V_c - m_c]\,[V_s - m_s]\,/\,H_i\,\} = 0 \qquad (3.9)$$

Thus, V_c and V_s are uncorrelated. Since they are Gaussian, V_c and V_s conditioned on any hypothesis are independent.

Using standard double random variable transformation techniques , we can obtain the conditional joint probability density function (p.d.f.) of V and α as a function of the conditional joint p.d.f. of V_c and V_s. Leaving out the mathematical details, we obtain

$$p(V, \alpha\,/H_i) = \frac{V}{2\pi\sigma^2}\exp\left\{-\frac{1}{2}\left[\frac{V^2}{\sigma^2} - 2\,\frac{V}{\sigma^2}\,(m_c\cos\alpha + m_s\sin\alpha) + \frac{m_c^2 + m_s^2}{\sigma^2}\right]\right\} \qquad (3.10)$$

for $0 \le V < \infty$ and $0 \le \alpha \le 2\pi$, $i=0,1,\ldots,M-1$

Introducing now the following change of variables

$$m_c = N\cos\mu \qquad (3.11)$$

$$m_s = N\sin\mu \qquad (3.12)$$

so that

$$N^2 = m_c^2 + m_s^2 = \frac{A^2}{2}\,(t_f - t_o)\,[(s_{i1} + J_1)^2 + (s_{i2} + J_2)^2] \qquad (3.13)$$

$$\frac{N^2}{\sigma^2} = \frac{(s_{i1} + J_1)^2 + (s_{i2} + J_2)^2}{N_o/2} \qquad (3.14)$$

and

$$\mu = \tan^{-1} \; (m_s/m_c)$$

$$= \tan^{-1}\left(\frac{s_{i2} + J_2}{s_{i1} + J_1}\right) \qquad (3.15)$$

it is possible to express Equation 3.10 in a different form. Observe that

$$m_c \cos\alpha + m_s \sin\alpha = N \cos(\alpha - \mu) \qquad (3.16)$$

so that we can write Equation 3.10 as

$$p(V, \; / H_i) = \begin{cases} \dfrac{V}{2\pi\sigma^2} \exp\left\{- \dfrac{1}{2}\left[\dfrac{V^2}{\sigma^2} - 2\dfrac{V}{\sigma^2} N \cos(\alpha - \mu) + \dfrac{N^2}{\sigma^2}\right]\right\} & \begin{matrix} 0 \leq V < \infty \\ 0 \leq \alpha \leq 2\pi \end{matrix} \\ 0 & \text{otherwise} \end{cases} \qquad (3.17)$$

In order to obtain the p.d.f. of α conditioned on H_i, we integrate $p\ (V,\alpha/H_i)$ over the range $0 \leq V < \infty$ and obtain,

$$p(\alpha / H_i) = \int_0^\infty \frac{V}{2\pi\sigma^2} \exp\left\{- \frac{1}{2}\left[\frac{V^2}{\sigma^2} - 2\frac{V}{\sigma^2} N \cos(\alpha - \mu) + \frac{N^2}{\sigma^2}\right]\right\} dV \qquad (3.18)$$

With the change of variables

$$r = V/\sigma \tag{3.19}$$

$p(\alpha/H_i)$ takes on the form

$$p(\alpha/H_i) = \int_0^{\infty} \frac{r}{2\pi} \exp\left\{-\frac{1}{2}\left[r^2 - 2r\frac{N}{\sigma}\cos(\alpha-\mu)+\frac{N^2}{\sigma^2}\right]\right\} dr \tag{3.20}$$

Returning to the receiver decision rule (see Equation 2.27), if $s_m(t)$ was transmitted, q_m is maximum if the distance $|\theta_m + \alpha|$ is minimum. That is, if

$$|\theta_m + \alpha| < |\theta_i + \alpha| \quad \text{for all } i \neq m \quad i=0,1,\ldots,M-1 \tag{3.21}$$

It can be seen that this inequality is valid for α in the region

$$-\theta_m - \pi/M < \alpha < -\theta_m + \pi/M \tag{3.22}$$

Thus, the probability of correctly detecting the mth signal is equivalent to the probability that α is in the region specified by the inequality of Equation 3.22. That is

$$P\{\text{correct decision }/H_m\} = \int_{-\theta_m-\pi/M}^{-\theta_m+\pi/M} p(\alpha/H_m)\, d\alpha$$

$$= \int_{-\theta_m-\pi/M}^{-\theta_m+\pi/M} \int_0^{\infty} \frac{r}{2\pi} \exp\left\{-\frac{1}{2}\left[r^2 - 2r\frac{N}{\sigma}\cos(\alpha-\mu) + \frac{N^2}{\sigma^2}\right]\right\} dr\, d\alpha \tag{3.23}$$

Letting

$$\beta = \alpha + \theta_m \tag{3.24}$$

then

$$P\{\text{correct decision}/H_m\} = \int_{-\pi/M}^{\pi/M} \int_0^{\infty} \frac{r}{2\pi} \exp\left\{ -\frac{1}{2}\left[r^2 - 2r\frac{N}{\sigma}\cos(\beta - \theta_m - \mu) + \frac{N^2}{\sigma^2} \right] \right\} dr\,d\beta \tag{3.25}$$

With the assumption that all signals are equally probable, the average probability of correct reception is

$$P\{\text{correct decision}\} = \frac{1}{M}\sum_{m=0}^{M-1} P\{\text{correct decision}/H_m\} \tag{3.26}$$

Hence, the M-ary PSK receiver word error probability becomes

$$P_e = 1 - P\{\text{correct decision}\}$$

$$= 1 - \frac{1}{M}\sum_{m=0}^{M-1} \int_{-\pi/M}^{\pi/M} \int_0^{\infty} \frac{r}{2\pi} \exp\left\{ -\frac{1}{2}\left[r^2 - 2r\frac{N}{\sigma}\cos(\beta - \theta_m - \mu) + \frac{N^2}{\sigma^2} \right] \right\} dr\,d\beta \tag{3.27}$$

Observe that in the absence of jamming

$$J_1 = J_2 = 0 \tag{3.28}$$

and

$$\frac{N^2}{\sigma^2} = SNR \quad ; \quad \mu = -\theta_m \tag{3.29}$$

so that the double integral becomes independent of the index m and therefore we obtain the same mathematical expression as in Equation 2.42 for P_e.

From the equation of probability of word error, it does not appear possible to analytically determine a jamming signal J(t) which is optimum in the sense of producing a maximum probability of error, subject to some "size" constraint on J(t). Observe that as $J(t) \rightarrow \infty$, $P_e \longrightarrow 1$. However it appears possible to postulate jammers that may have near optimum qualities in the sense defined above.

1. Weighted Sum Of The Signals Jammer

An initial jammer choice is one in which J(t) is made up of a weighted sum of the signals $s_i(t)$. For simplicity, uniform weights are chosen. The jammer power is denoted P_j, so that J(t) takes on the form

$$J(t) = \sqrt{2 P_j/M} \sum_{i=0}^{M-1} \cos (w_c t + 2\pi i/M) \tag{3.30}$$

Observe that

$$J(t) = \sqrt{2 \ P_j/M} \ . \ \frac{1}{A} \sum_{i=0}^{M-1} s_i(t)$$

$$= \sqrt{2 \ Pj/M} \ . \ \frac{1}{A} \sum_{i=0}^{M-1} [\ s_{i1}\ g_1(t) + s_{i2}\ g_2(t)\ ] \tag{3.31}$$

From Equation 3.14 and Equation 3.15 we see that performance of the receiver is affected by the components of the jamming signal in the direction of the orthonormal basis functions $g_1(t)$ and $g_2(t)$. Thus it can be shown that for this jammer choice

$$J_1 = \int_{t_o}^{t_f} J(t)\, g_1(t)\, dt$$

$$= \sqrt{P_j\, (t_f - t_o)/M} \left[1 - \frac{\sin \pi\ (1-1/M)}{\sin \pi / M} \right] = 0 \tag{3.32}$$

and

$$J_2 = \int_{t_o}^{t_f} J(t)\, g_2(t)\, dt$$

$$= -\sqrt{P_j\, (t_f - t_o)/M} \left[\frac{\sin(M-2\pi/2M)\ .\ \sin((M-1)2\pi/2M)}{\sin 2\pi /M} \right]$$

$$= 0 \tag{3.33}$$

Since J_1 and J_2 are equal to zero, Equations 3.14 and 3.15 are equivalent to the statement of Equation 3.29. Therefore we obtain the same result for word error probability of the M-PSK receiver in white Gaussian noise only interference as given in Equation 2.42.

This surprising result can be analyzed from a different perspective. From Equation 2.26, in the presence of noise and jamming, the output q_i of each channel of the simplified version for M-PSK of the receiver of Figure 1 given that $s_m(t)$ was transmitted, is given by

$$q_i = \int_{t_0}^{t_f} [s_m(t) + n(t) + J(t)]\, s_i(t)dt \qquad \begin{matrix} i=0,1,\ldots,M-1 \\ m=0,1,\ldots,M-1 \end{matrix} \qquad (3.34)$$

The receiver is now affected in each channel due to the component of the jamming waveform J(t) in the direction of the signal $s_i(t)$ for $i=0,1,\ldots,M-1$. That is, the jamming component of q_i, is

$$\int_{t_0}^{t_f} J(t)\, s_i(t)\, dt \qquad i=0,1,\ldots,M-1$$

With the choice of a uniformly weighted sum of signals jammer, using the first equality of Equation 3.31 and Equation 2.24, the jamming component of q_i becomes

$$\int_{t_0}^{t_f} J(t)\, s_i(t)\, dt = \sqrt{2\, P_j/M}\;\; A(t_f - t_o)/2 \quad \sum_{j=o}^{M-1} \overline{\rho}_{ij}$$

$$i=0,1,\ldots,M-1 \qquad (3.35)$$

It can be shown without difficulty that $\sum_{j=0}^{M-1} \overline{\rho}_{ij}$ is zero for $i=0,1,\ldots,M-1$. Thus the jamming component of q_i is zero for all M channels of the receiver. Therefore the uniform weighted sum of signals jammer has no effect on the M-PSK receiver performance, and would as a result of this be a poor choice for a jamming waveform. A non-uniformly weighted sum of signals may prove to be a better jammer however this has not been analyzed in this thesis.

2. Pure Tone Jammer at Carrier Frequency With Set Phase

The jammer choice here is such that J(t) is given by

$$J(t) = \sqrt{2 P_j} \cos (w_c t + 2\pi i/M) \qquad (3.36)$$

for some $i = 0, 1, \ldots, M-1$. This proposed jammer has the following components J_1 and J_2 .

$$J_1 = \sqrt{2 P_j (t_f - t_o)/2} \cos 2\pi i/M \qquad (3.37)$$

and

$$J_2 = -\sqrt{2P_j (t_f - t_o)/2} \sin 2\pi i/M \qquad (3.38)$$

If we define the jamming to signal ratio (JSR) as

$$JSR = P_j / (A^2/2) \qquad (3.39)$$

and with the definition of SNR in Equation 2.46, it can be shown that Equation 3.14 and Equation 3.15 become respectively

$$\frac{N^2}{\sigma^2} = SNR [1+2\sqrt{JSR} \cos(\theta_m - \theta_i) + JSR] \qquad (3.40)$$

and

$$\mu = -\tan^{-1}\left(\frac{\sin\theta_m + \sqrt{JSR} \sin\theta_i}{\cos\theta_m + \sqrt{JSR} \cos\theta_i}\right) \qquad (3.41)$$

A computer program has been written to evaluate P_e given by Equation 3.27 using Equation 3.40 and Equation 3.41 in order to obtain numerical results that will quantify the effectiveness of this jammer.

Word error probability versus signal to noise ratio is plotted and shown in Figures 4, 5, 6 and 7 for BPSK (M = 2), QPSK (M = 4), 8PSK (M = 8) and 16PSK (M = 16) respectively, for different jamming to signal ratios.

B. PERFORMANCE OF THE M-ASK RECEIVER IN JAMMING

In a jamming environment of the type analyzed in the previous section, the output of each channel of the receiver of Figure 3 becomes

$$q_i = \left[\int_{t_0}^{t_f} [s_i(t) + n(t) + J(t)]\, g_1(t)\, dt \right] A_i \sqrt{E_f} - A_i^2 E_f/2 \qquad i=0,1,\ldots,M-1 \qquad (3.42)$$

As before, the decision rule implemented by the receiver is to choose H_j as the true hypothesis if q_j is the largest, that is, if

$$q_j > q_0,\quad q_j > q_1,\ \ldots,\ q_j > q_{j-1},\quad q_j > q_{j+1},\ \ldots,\quad q_j > q_{M-1} \qquad (3.43)$$

If $s_j(t)$ is transmitted, then Equation 3.42 becomes

$$q_i = A_i \sqrt{E_f}\,(A_j \sqrt{E_f} + n_1 + J_1) - A_i^2 E_f/2 \qquad (3.44)$$

$$i=0,1,\ldots,M-1$$
$$j=0,1,\ldots,M-1$$

Assuming that the M possible signals $s_i(t)$ are ordered in increasing amplitude, that is

$$A_0 < A_1 < A_2 \ldots < A_{j-1} < A_j < A_{j+1} < \ldots < A_{M-1} \qquad (3.45)$$

using Equation 3.44 in the inequalities of Equation 3.43 leads to the observation that given that $s_j(t)$ was transmitted, no error is made if n_1, the noise component in the direction of $g_1(t)$, ranges between the limits of the following inequality

$$-\frac{1}{2}(A_j - A_{j-1})\sqrt{E_f} - J_1 < n_1 < \frac{1}{2}(A_{j+1} - A_j)\sqrt{E_f} - J_1 \qquad (3.46)$$

Observe that n_1 is a conditional Gaussian random variable with expected value

$$E\{ n_1 / H_j \} = 0 \qquad (3.47)$$

(due to the zero mean noise assumption) and variance

$$\mathrm{Var}\{ n_1 / H_j \} = N_o/2 \qquad (3.48)$$

Therefore the probability density function of n_1 conditioned on H_j is

$$p(n_1/H_j) = \frac{1}{\sqrt{2\pi(N_o/2)}} \exp[-n_1^2 / 2(N_o/2)] \qquad (3.49)$$

Letting $C_{j1\pm}$ denote $\frac{1}{2}(A_{j\pm1} - A_j)\sqrt{E_f} - J_1$, the probability of correctly detecting the jth signal is given by

$$P\{\text{no error} / H_j\} = \int_{C_{j1-}}^{C_{j1+}} \frac{1}{\sqrt{2\pi(N_o/2)}} \exp[-n_1^2 / 2(N_o/2)]dn_1 \tag{3.50}$$

With a change of variables Equation 3.50 takes on the form

$$P\{\text{no error} / H_j\} = \int_{C_{j1-}/\sqrt{N_o/2}}^{C_{j1+}/\sqrt{N_o/2}} \frac{1}{\sqrt{2\pi}} \exp[-x^2/2]\,dx \tag{3.51}$$

If we assume now that the separation between each pair of consecutive signals is constant, that is

$$A_{j+1} - A_j = A = A_j - A_{j-1} \qquad j=1,\ldots,M-1 \tag{3.52}$$

then the probability of correctly detecting the jth signal is independent of the index j and the receiver average probability of correct detection is

$$P\{\text{no error}\} = P\{\text{no error} / H_j\}$$

$$= \int_{C-}^{C+} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx \tag{3.53}$$

where C ± denotes $[\pm \frac{1}{2} A\sqrt{E_f} - J_1] / \sqrt{N_o/2}$. Thus the word error probability of the M-ASK receiver in the presence of jamming becomes

$$P_e = 1 - P \{ \text{no error} \}$$

$$= 1 - \int_{C-}^{C+} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \qquad (3.54)$$

$$= \text{erf}_*(C-) + \text{erfc}_*(C+)$$

It is worth noting that if $J_1 = 0$, then the last expression for P_e in Equation 3.54 becomes identical to the expression for P_e obtained in Equation 2.51.

From the second equality in Equation 3.54 involving the M-ASK receiver probability of error, the effect of the jamming waveform on performance can be analyzed by investigating the derivative of the receiver probability of error with respect to J_1. Carrying this out we obtain

$$\frac{\partial P_e}{\partial J_1} = \frac{1}{\sqrt{\pi N_o}} e^{-\frac{\frac{1}{4} A^2 E_f + J_1^2}{N_o}} .2 \sinh \left(\frac{J_1 A E_f}{N_o} \right) \qquad (3.55)$$

From Equation 3.55, it follows that

$$\frac{\partial P_e}{\partial J_1} = \begin{cases} > 0 & J_1 > 0 \\ = 0 & J_1 = 0 \\ < 0 & J_1 < 0 \end{cases} \tag{3.56}$$

Thus P_e is monotonic in J_1. It is a decreasing function for negative values of J_1 and is an increasing function for positive values of J_1. Therefore we can maximize P_e by making J_1 as large in magnitude as possible.

From the Cauchy-Schwarz inequality we have that

$$J_1 = \int_{t_o}^{t_f} J(t) g_1(t) dt \leq \left[\int_{t_o}^{t_f} J^2(t) dt\right]^{1/2} \cdot \left[\int_{t_o}^{t_f} {g_1}^2(t) dt\right]^{1/2} \tag{3.57}$$

thus we conclude that J_1 can be only as large as the square root of the energy of the jamming waveform and this will occur if and only if

$$J(t) = K\, g_1(t) = K \frac{f(t)}{\sqrt{E_f}} \tag{3.58}$$

Since we normally must satisfy some energy constraint on the jammer, that is

$$\int_{t_o}^{t_f} J^2(t) dt = \int_{t_o}^{t_f} K^2 \frac{f^2(t)}{E_f} dt = K^2 \tag{3.59}$$

it can be seen that the maximum value of J_1 is

$$J_1 = K \tag{3.60}$$

and the maximum P_e that can be obtained with such a constrained jammer is

$$P_e = 1 - \int_{\left[-\frac{1}{2} A\sqrt{E_f} - K\right] / \sqrt{N_o/2}}^{\left[\frac{1}{2} A\sqrt{E_f} - K\right] / \sqrt{N_o/2}} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \qquad (3.61)$$

Observe that we can interpret signal to noise ratio (SNR) and jammer to signal ratio (JSR) as

$$\frac{\frac{1}{2} A\sqrt{E_f}}{\sqrt{N_o/2}} = \frac{1}{2}\sqrt{\frac{A^2 E_f}{N_o/2}} = \sqrt{SNR} \;/2 \qquad (3.62)$$

and

$$\frac{K}{\sqrt{N_o/2}} = \sqrt{\frac{K^2}{N_o/2}} = \sqrt{JSR} \qquad (3.63)$$

Using these definitions, the M-ASK receiver word error probability with optimum jamming becomes

$$P_e = \mathrm{erf}_* \left(-\frac{1}{2}\sqrt{SNR} - \sqrt{JSR}\right) + \mathrm{erfc}_* \left(\frac{1}{2}\sqrt{SNR} - \sqrt{JSR}\right) \qquad (3.64)$$

We can see that for a fixed JSR, increasing SNR will cause the receiver probability of error to tend to zero. A computer program has been written to evaluate P_e given by

Equation 3.64. Performance versus SNR is plotted and shown in Figure 8 for a set of values of JSR.

IV. DESCRIPTION OF GRAPHICAL RESULTS

In this chapter, plots resulting from evaluation of the derived performance expressions for the M-PSK and M-ASK receivers are presented and interpreted. These plots are useful in quantifying the effectiveness of the proposed jamming waveforms. The graphical results display receiver word error probability as a function of signal to noise ratio (SNR) for different values of jamming to signal ratio (JSR) for the specified jammer. Each plot presents the case of JSR = 0.0 in order to allow comparisons of the jammer effectiveness to the receiver performance operating in additive white Gaussian noise only interference.

A. M-PSK RECEIVER PERFORMANCE GRAPHICAL RESULTS

Graphical results of receiver word error probability for BPSK, QPSK, 8PSK and 16PSK modulation were obtained for the case in which a tone jammer with fixed phase was used.

Computer evaluation of Equation 3.27 using Equations 3.40 and 3.41 was undertaken, and the corresponding graphical results are shown in Figures 4, 5, 6 and 7 for BPSK, QPSK, 8PSK and 16PSK modulation respectively. We can observe in all these plots the "breakpoint" phenomenon [Ref. 7]. That is, for JSR beyond a specific value, P_e does not monotonically decay with increasing SNR. This can be explained as follows. As the power of the jamming waveform increases, the channel corresponding to the signal being used as a jammer becomes larger also. This means that the receiver decides most of the time in favor of the signal in the jammed channel, while the probability of such a signal being transmitted remains at 1/M. Thus the probability that the receiver will err becomes 1 - 1/M, which approaches 1 for moderate values of M.

For BPSK modulation (M=2), from Figure 4 we can see that as JSR takes on values greater than or equal to 1, P_e tends to 1/2 with increasing SNR. Note that in order to obtain P_e of 10^{-6}, 13.5 dB of SNR are required at a JSR of 0.0, 16.5 dB of SNR are required at a JSR of 0.1, and 23.8 dB of SNR are required at a JSR of 0.5. Observe also that for JSR > 1, it is not possible to obtain P_e of 10^{-6}.

For QPSK modulation (M = 4), from Figure 5, we note that as JSR takes on values greater than or equal to 1, P_e tends to 3/4 as SNR increases. In order to obtain a P_e of 10^{-6}, 16.8 dB of SNR are required at a JSR of 0.0, 19.5 dB of SNR are required at a JSR of 0.1 and 27.2 dB of SNR are required at a JSR of 0.5.

For 8PSK modulation (M=8), from Figure 6, we notice that as JSR takes on values greater than or equal 0.5, P_e tends to 7/8 with increasing SNR. In order to obtain P_e of 10^{-6}, 21.9 dB of SNR are required at a JSR of 0.0 and 34 dB of SNR are required at a JSR of 0.1.

Finally, from Figure 7, for 16PSK modulation (M=16), we can see that the tone jammer renders the receiver inoperative since P_e tends to 15/16 as JSR takes on values greater than zero. In order to obtain P_e of 10^{-6}, 27.9 dB of SNR are required at JSR = 0.0. Certainly, as the number of levels (M) of the signals increases, the probability of word error increases. This behavior is well known. Consequently, it is not surprising that relatively low JSR values can render the receiver effectively inoperative.

The results of Chapter 3 demonstrated that the uniform weighted sum of signals jammer has no effect on the M-PSK receiver performance. For this reason, no plots are presented for this particular case.

B. M-ASK RECEIVER PERFORMANCE GRAPHICAL RESULTS

Graphical results on the performance of the M-ASK receiver were obtained for the optimum power constrained jamming waveform derived in Chapter 3. Through computer evaluation of Equation 3.64, graphical results are shown in Figure 8 for receiver word error probability as a function of SNR for fixed JSR. Observe that P_e tends to zero as SNR increases for a fixed but arbitrary value of JSR. There is no "breakpoint" phenomenon here because this M-ASK receiver is not influenced by a threshold. A better understanding of this comes from a re-examination of Equation 3.44. It demonstrates that the ratio of jammer power in any one channel to jammer power in any other channel remains unchanged as jammer power increases or decreases. Note that in order to obtain P_e of 10^{-6}, 19.8 dB of SNR are required for a JSR = 0.0, 20.1 dB of SNR are required for a JSR = 0.1, 20.6 dB of SNR are required for a JSR = 0.5 and 22.9 dB of SNR are required for a JSR = 5.0. Observe furthermore that the results are independent of the number M of signals used. This is due to the assumption that signal separation was constant. Therefore as M increases, the average energy (and peak energy) of the signal set must increase (assuming equally likely signals). Without a constraint on average or peak energy, results will be independent of M. It is possible to derive results in jamming effect for peak energy constrained signal sets. As can be expected under these conditions, the receiver performance worsens as M increases.

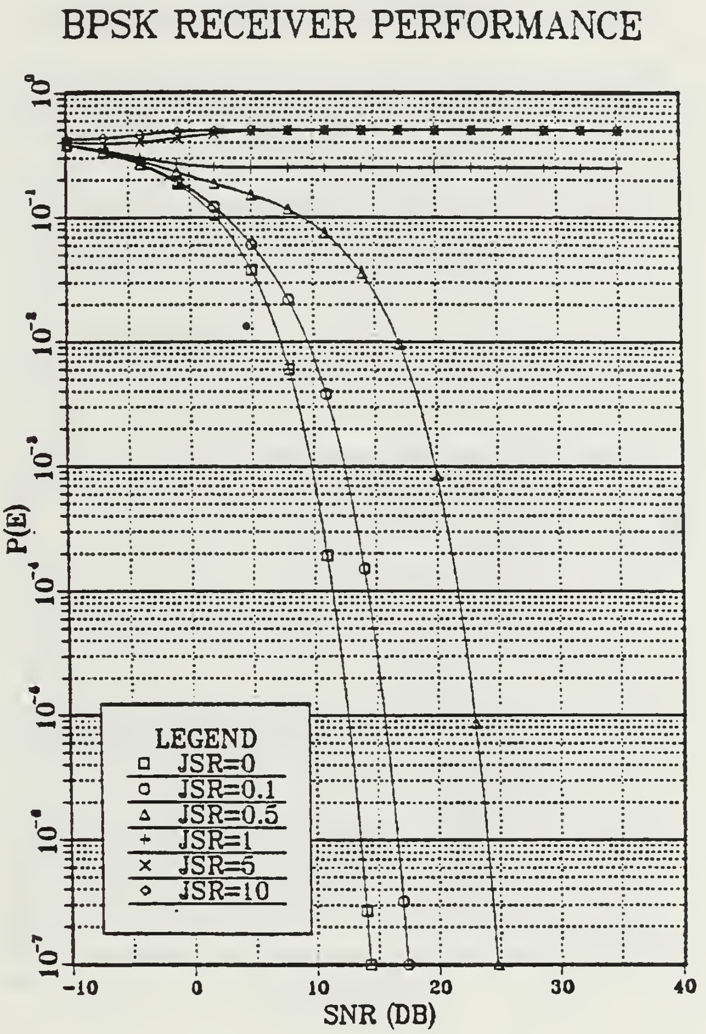

Figure 4 BPSK Receiver Performance with Tone Jamming

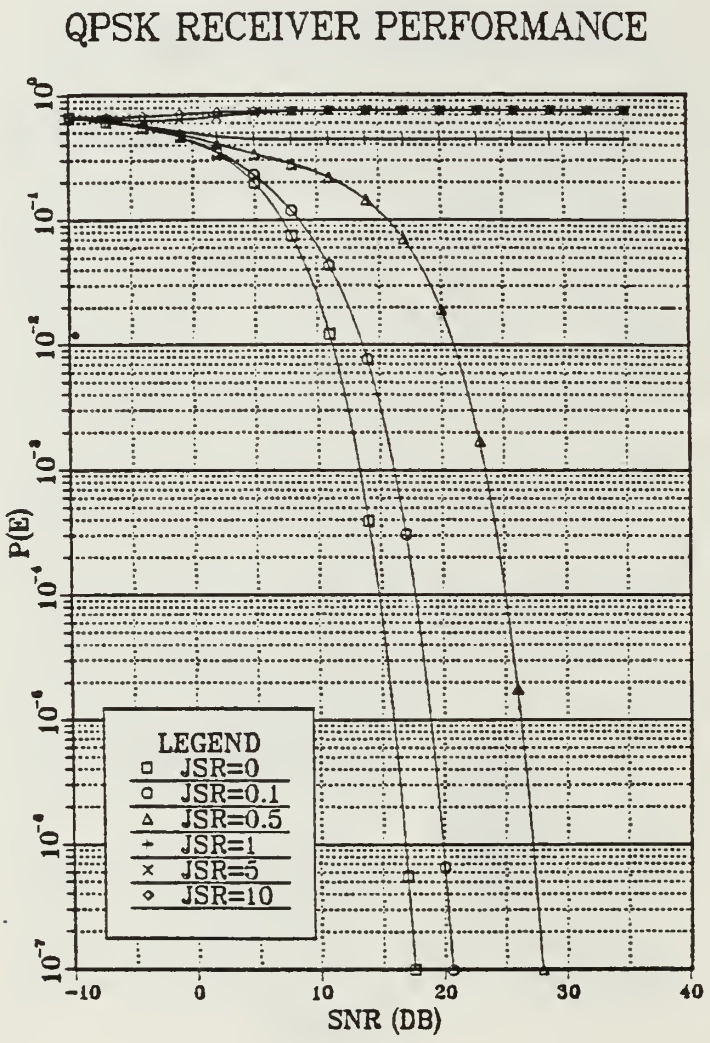

Figure 5 QPSK Receiver Performance With Tone Jamming

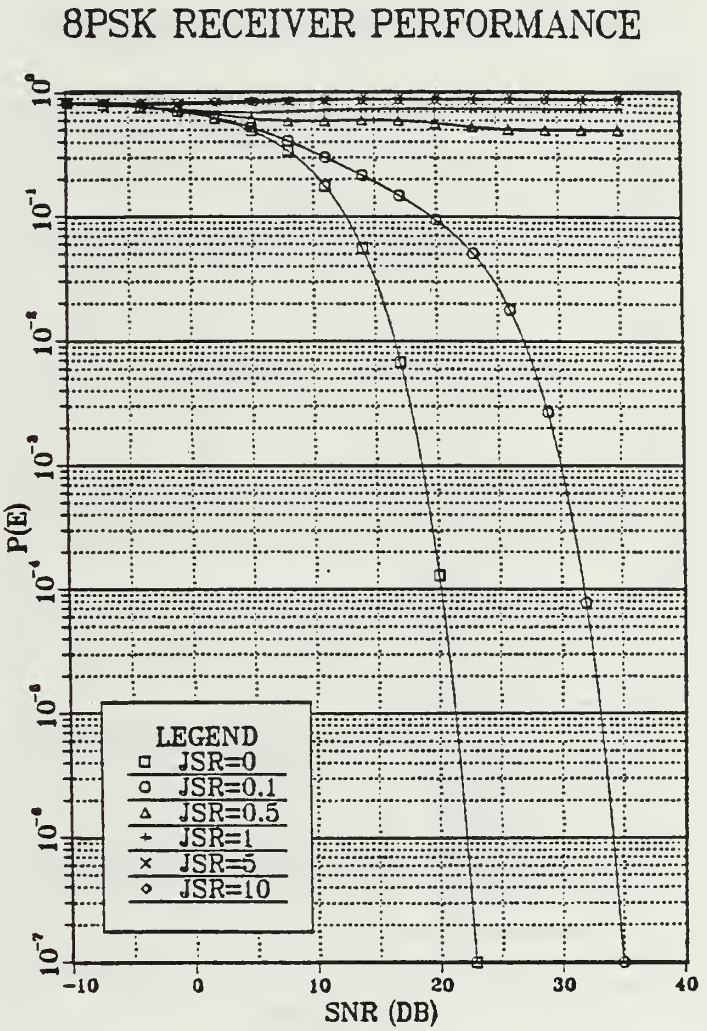

Figure 6 8PSK Receiver Performance With Tone Jamming

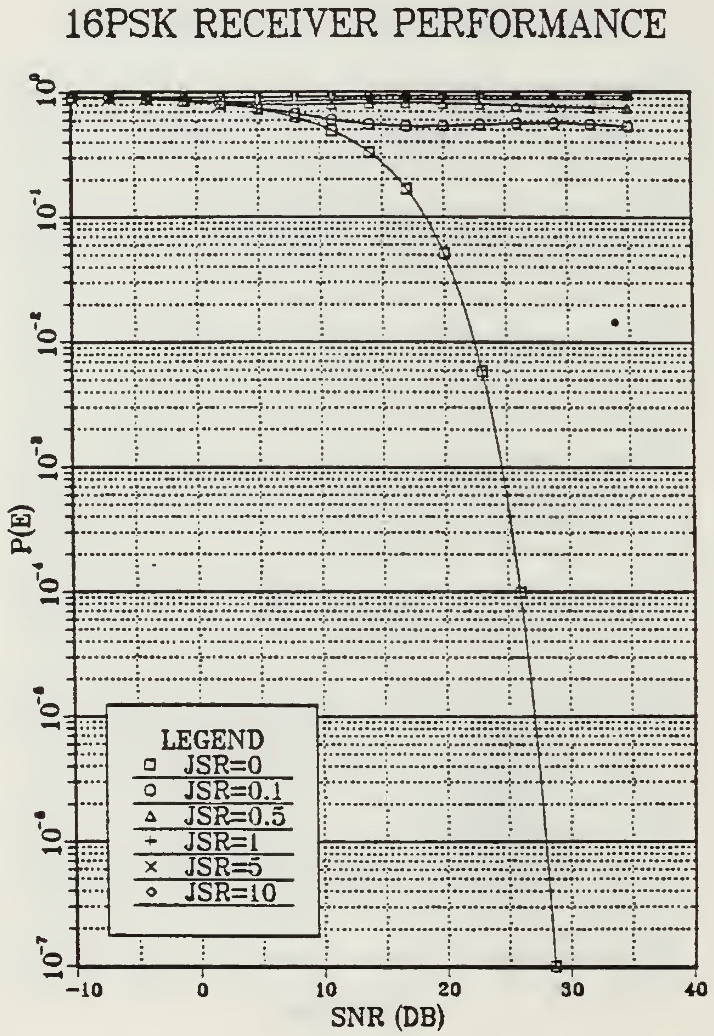

Figure 7 16PSK Receiver Performance With Tone Jamming

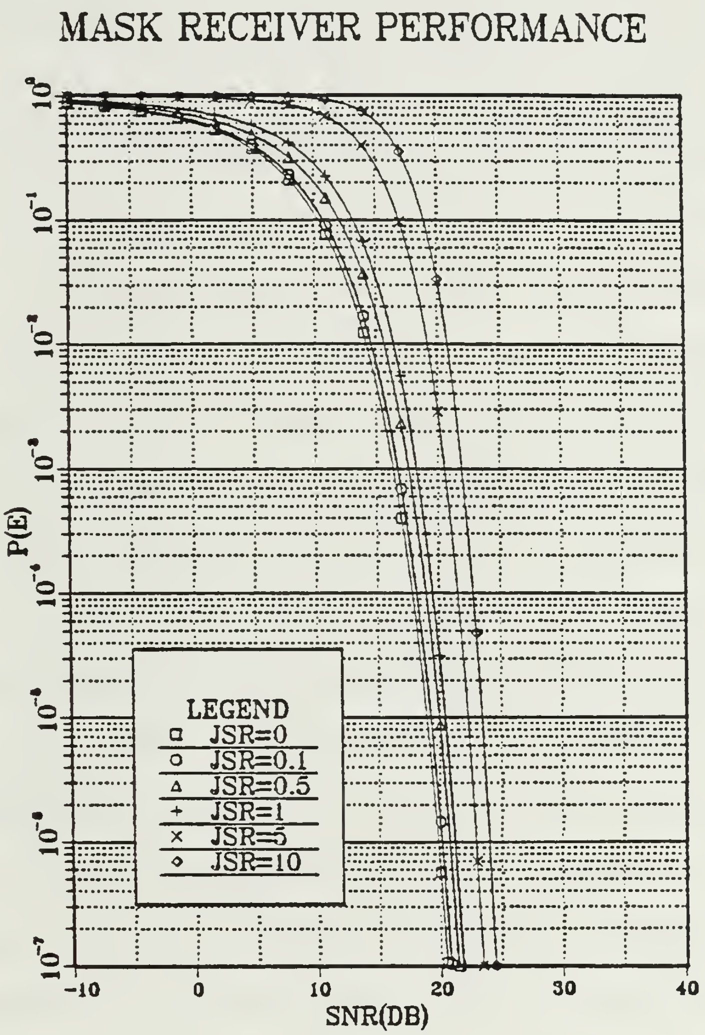

Figure 8 M-ASK Receiver Performance With Optimum Jamming

V. CONCLUSION

The optimum coherent receiver for multilevel digital signals designed to operate in additive white Gaussian noise environment was analyzed for the special cases of M-PSK and M-ASK modulation under the presence of jamming waveforms. Receiver word error probability was used as a measure of receiver performance. Jammer waveforms capable of degrading receiver performance were postulated and analyzed. For M-PSK, it is concluded that a uniformly weighted sum of signals jammer has no effect on the receiver performance. Using a tone jammer at the carrier frequency with set phase, it was concluded that for a moderate number of signals, relatively low jammer to signal ratios (JSR) render the M-PSK receiver effectively inoperative. The optimum jamming waveform, optimum in the sense of producing maximum receiver probability of error for a given jammer power level, was derived for M-ASK modulation where the ASK signals have constant separation. (No constraint on average or peak energy of the signals was used). For equally probable signals the optimum power constrained jammer was derived as specified in Equation 3.58.

For future analysis, it is suggested to consider other jamming strategies that may prove to be effective. For the case of M-PSK modulation a good jammer choice could prove to be a non uniform weighted sum of signals, or a sum or difference of a pair of signals. For the case of M-ASK modulation it is suggested to derive results on jamming effects for peak energy and average energy constrained signal sets. Under these conditions it can be expected that the receiver performance worsens as M increases.

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